Section 4-2 Node Voltage Analysis of Circuits with Current Sources

P 4.2-1 The node voltages in the circuit of Figure P 4.2-1 are

$v_1 = -4$ V and $v_2 = 2$ V.

Determine i, the current of the current source.
Answer: $i = 1.5$ A

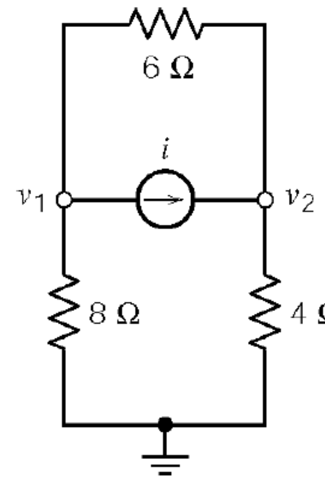


Figure P 4.2-1

Solution:

KCL at node 1: $$0 = \frac{v_1}{8} + \frac{v_1 - v_2}{6} + i = \frac{-4}{8} + \frac{-4-2}{6} + i = -1.5 + i \quad \Rightarrow \quad i = 1.5 \text{ A}$$

P 4.2-2 Determine the node voltages for the circuit of Figure P 4.2-2.

Answer: $v_1 = 2$ V, $v_2 = 30$ V, and $v_3 = 24$ V

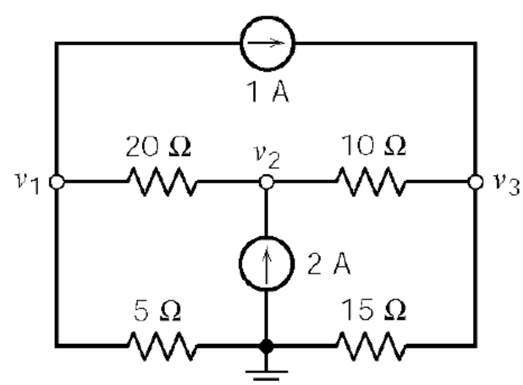


Figure P 4.2-2

Solution:

KCL at node 1: $$\frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = 0 \quad \Rightarrow \quad 5\,v_1 - v_2 = -20$$

KCL at node 2: $$\frac{v_1 - v_2}{20} + 2 = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad -v_1 + 3\,v_2 - 2\,v_3 = 40$$

KCL at node 3: $$\frac{v_2 - v_3}{10} + 1 = \frac{v_3}{15} \quad \Rightarrow \quad -3\,v_2 + 5\,v_3 = 30$$

Solving gives $v_1 = 2$ V, $v_2 = 30$ V and $v_3 = 24$ V.

Section 4-3 Node Voltage Analysis of Circuits with Current and Voltage Sources

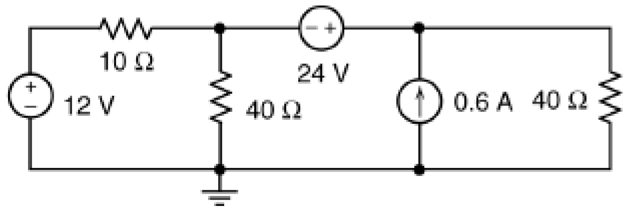
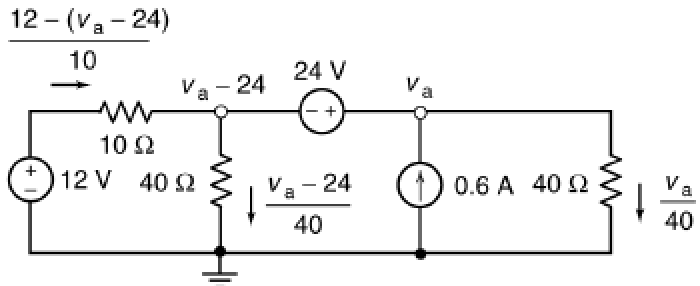


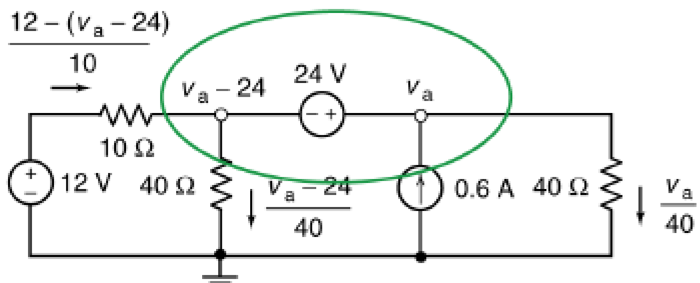
Figure P4.3-3

P4.3-3. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-3.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 24 V source



Apply KCL to the supernode to get

$$\frac{12-(v_a-24)}{10}+0.6=\frac{v_a-24}{40}+\frac{v_a}{40} \quad \Rightarrow \quad 196=6v_a \quad \Rightarrow \quad v_a=32 \text{ V}$$

The 12 V source supplies $12\left(\frac{12-(v_a-24)}{10}\right)=12\left(\frac{12-(32-24)}{10}\right)=4.8 \text{ W}$

The 24 V source supplies $24\left(-0.6+\frac{v_a}{40}\right)=24\left(-0.6+\frac{32}{40}\right)=4.8 \text{ W}$

The current source supplies $0.6v_a=0.6(32)=19.2 \text{ W}$

Section 4-4 Node Voltage Analysis with Dependent Sources

P 4.4-3 Determine the node voltage v_b for the circuit of Figure P 4.4-3.
Answer: $v_b = 1.5$ V

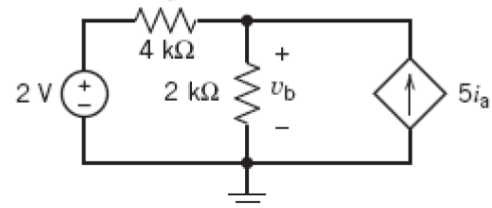
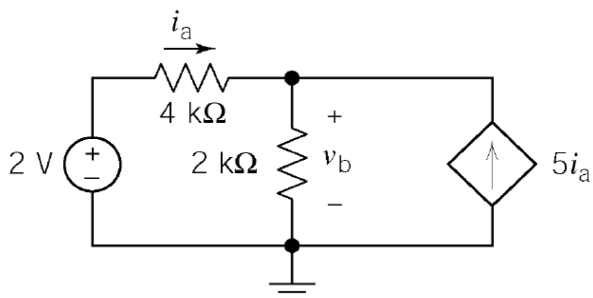


Figure P 4.4-3

Solution:



First express the controlling current in terms of the node voltages:

$$i_a = \frac{2 - v_b}{4000}$$

Write and solve a node equation:

$$-\frac{2 - v_b}{4000} + \frac{v_b}{2000} - 5\left(\frac{2 - v_b}{4000}\right) = 0 \quad \Rightarrow \quad \underline{v_b = 1.5 \text{ V}}$$

Section 4-5 Mesh Current Analysis with Independent Voltage Sources

P 4.5-1 Determine the mesh currents, i_1, i_2, and i_3, for the circuit shown in Figure P 4.5-1.

Answers: $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 4$ A

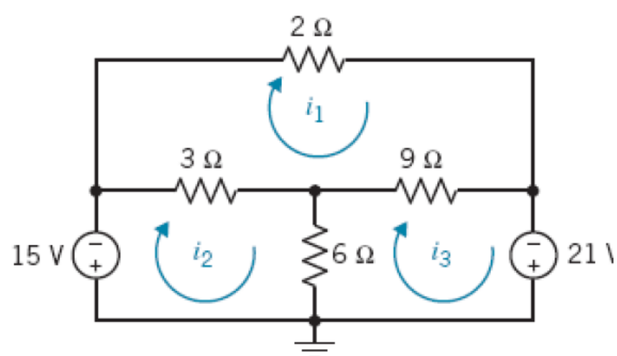


Figure P 4.5-1

Solution:
The mesh equations are

$$2\,i_1 + 9\,(i_1 - i_3) + 3(i_1 - i_2) = 0$$
$$15 - 3\,(i_1 - i_2) + 6\,(i_2 - i_3) = 0$$
$$-6\,(i_2 - i_3) - 9\,(i_1 - i_3) - 21 = 0$$

or

$$14\,i_1 - 3\,i_2 - 9\,i_3 = 0$$
$$-3\,i_1 + 9\,i_2 - 6\,i_3 = -15$$
$$-9\,i_1 - 6\,i_2 + 15\,i_3 = 21$$

so

$$i_1 = 3 \text{ A}, \;\; i_2 = 2 \text{ A and } i_3 = 4 \text{ A}.$$

Section 4-6 Mesh Current Analysis with Voltage and Current Sources

P 4.6-2 Find v_c for the circuit shown in Figure P 4.6-2.

Answer: $v_c = 15$ V

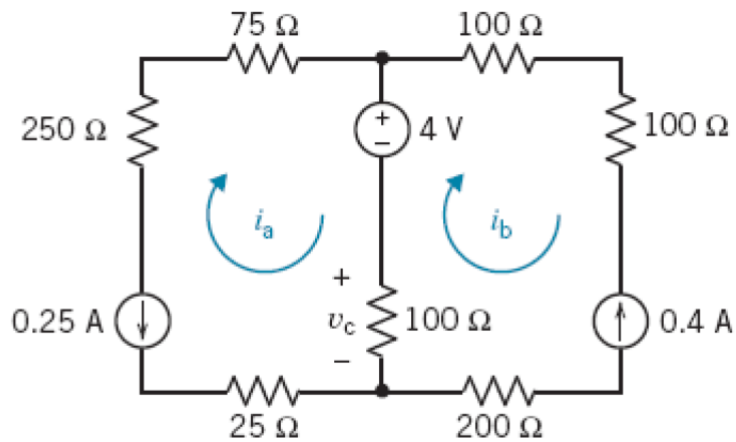


Figure P 4.6-2

Solution:

Mesh currents:

mesh a: $i_a = -0.25$ A

mesh b: $i_b = -0.4$ A

Ohm's Law:

$v_c = 100(i_a - i_b) = 100(0.15) = \underline{15\ \text{V}}$

P 4.6-4 Find v_c for the circuit shown in Figure P 4.6-4.

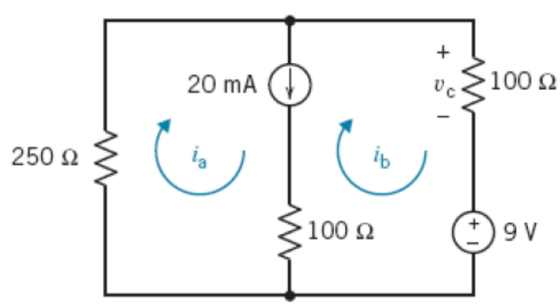


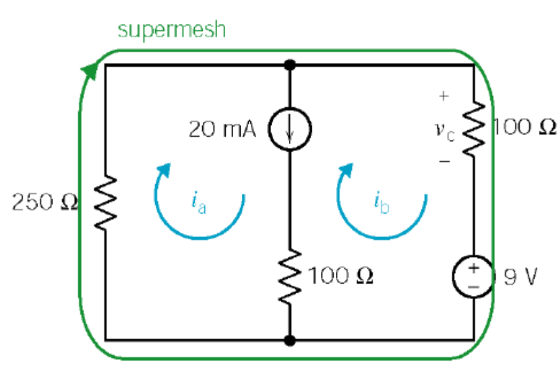
Figure P 4.6-4

Solution:

Express the current source current in terms of the mesh currents:

$$i_b = i_a - 0.02$$

Apply KVL to the supermesh:



$$250\, i_a + 100\,(i_a - 0.02) + 9 = 0$$

$$\therefore i_a = -.02 \text{ A} = -20 \text{ mA}$$

$$v_c = 100(i_a - 0.02) = \underline{-4\ \text{V}}$$

Section 4-7 Mesh Current Analysis with Dependent Sources

P 4.7-1 Find v_2 for the circuit shown in Figure P 4.7-1.

Answer: $v_2 = 10$ V

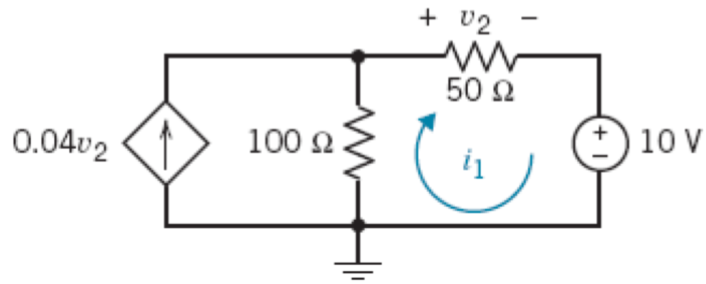


Figure P 4.7-1

Solution:

Express the controlling voltage of the dependent source as a function of the mesh current

$$v_2 = 50\, i_1$$

Apply KVL to the right mesh:

$$-100\,(0.04(50i_1) - i_1) + 50i_1 + 10 = 0 \Rightarrow i_1 = 0.2 \text{ A}$$

$$v_2 = 50\, i_1 = 10 \text{ V}$$

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

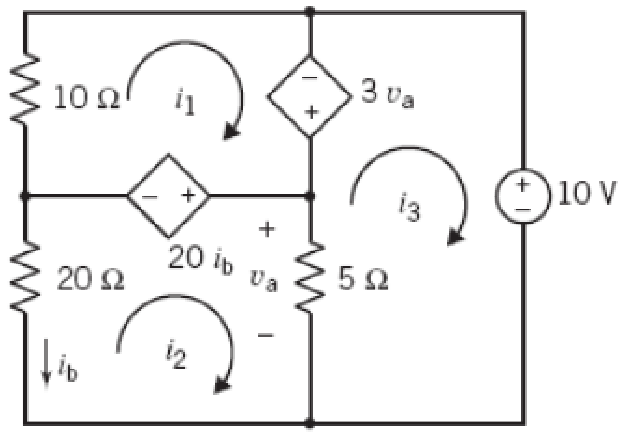


Figure P 4.7-7

Solution:

Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5(i_2 - i_3) \text{ and } i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\ i_b = -20\ i_2 \text{ and } 3\ v_a = 15(i_2 - i_3)$$

Apply KVL to the meshes

$$-15(i_2 - i_3) + (-20\ i_2) + 10\ i_1 = 0$$
$$-(-20\ i_2) + 5(i_2 - i_3) + 20\ i_2 = 0$$
$$10 - 5(i_2 - i_3) + 15\ (i_2 - i_3) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -1.25 \text{ A},\ i_2 = +0.125 \text{ A, and } i_3 = +1.125 \text{ A}$$